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Physics of Kinetic Alfvén Waves: A Gyrokinetic Theory Approach†

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Outlines

(I) Introduction: Alfvén wave continuum, phase mixing, "singular" resonance

(II) Linear Kinetic physics: Linear gyrokinetic theory, mode conversion, spatial/temporal scales, observations

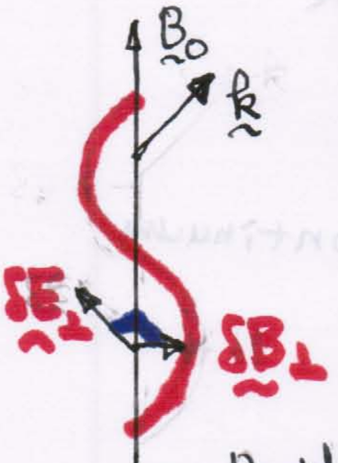
(III) Nonlinear Kinetic physics: Nonlinear gyrokinetic theory, parametric decay, convective cells, simulations

(IV) Summary & Discussions

† In collaborations with Y. Lin (Auburn U) and F. Zonca (ENEN Frascati)

(I) Introduction

◎ Shear Alfvén waves (SAW) [H. Alfvén 1942] ⇒ Anisotropic electromagnetic waves in magnetized plasmas (Laboratory, space, solar etc.): Anisotropic conducting medium [$\sigma_{\parallel} \to \infty$, $\sigma_{\perp}$: finite]



$$\boxed{\omega^2 = k_{\parallel}^2 V_A^2} \equiv \omega_A^2$$

$$\boxed{\underset{\sim}{V}_g = V_A \underset{\sim}{b}_0}$$

$$k_{\parallel} = \underset{\sim}{k} \cdot \underset{\sim}{B}_0 / B_0 \equiv \underset{\sim}{k} \cdot \underset{\sim}{b}_0$$

$$\boxed{V_A^2 = \left[4\pi n_0 m_i / B_0^2\right]^{-1}}$$

- Readily excited by either external perturbations (e.g. solar wind, antenna) or intrinsic instabilities (e.g. beam/rf injections)

- In realistic nonuniform plasmas –

$\omega_A^2 \Rightarrow \omega_A^2(x)$; x: (radial) nonunifomities

$$\Rightarrow \boxed{\omega^2 = \omega_A^2(x)} : \text{SAW continuous spectrum}$$

[first noted by H. Grad (1969)]

$$\Rightarrow \boxed{\left[\partial_t^2 + \omega_A^2(x)\right] \delta B_y(x,t) = 0}$$

- $\delta B_y(x,t) = \delta \hat{B}_y(x, t=0) \exp\left[-i\omega_A(x) t\right]$

AMPTE/CCE Satellite

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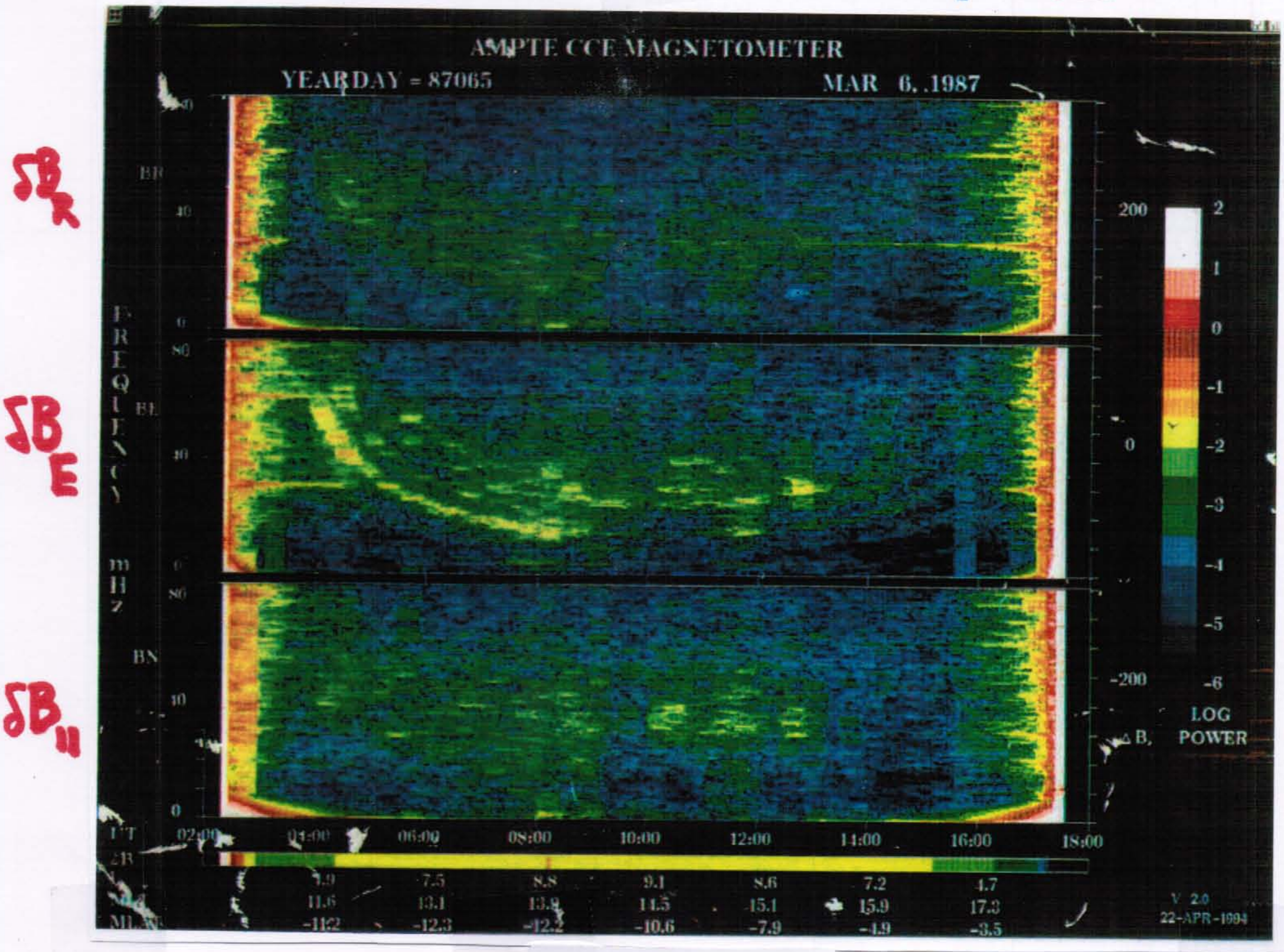
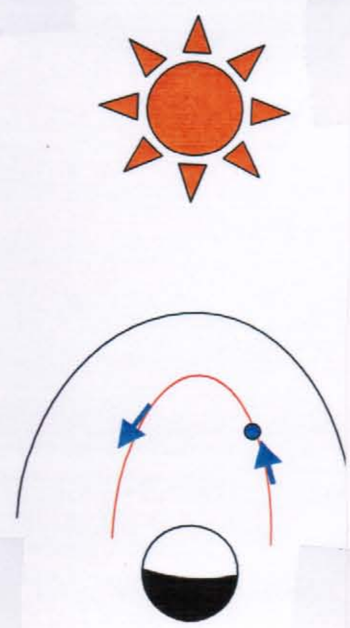
[Engebretson et al, 1987] JGR

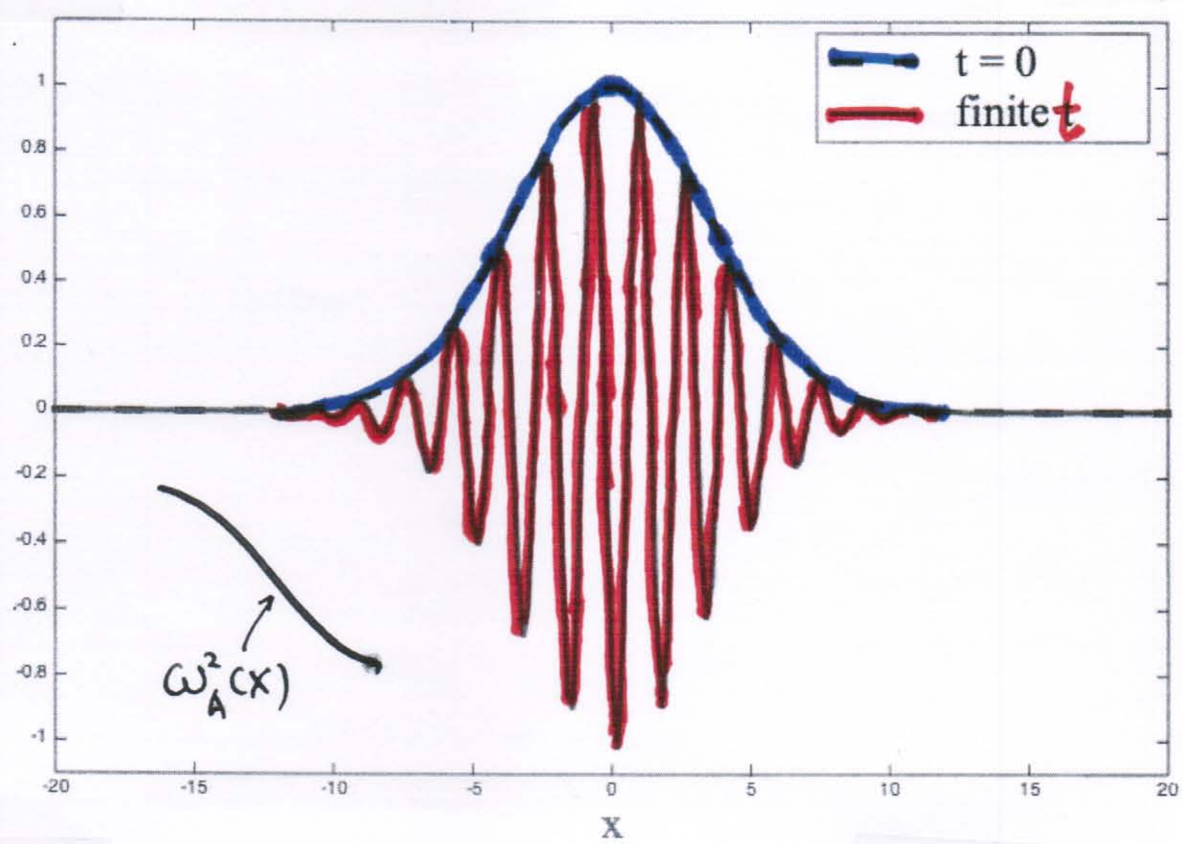




$\Rightarrow$ Significant implications to spatial scales:

$$|k_x| \equiv \left|\frac{d \ln \delta B_y}{dx}\right| \Rightarrow \left|\frac{d\omega_A}{dx}\right| t \quad \text{(phase mixing)}$$

- As t increases: long scales evolve into short scales



[Z. Qiu Thesis] GAM

$\Rightarrow$ $|k_x| \to \infty$ as $t \to \infty$: singularity at steady state

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◎ Steady state SAW equation :

$$\boxed{\left\{\frac{d}{dx}\left[\omega_A^2(x)-\omega^2\right]\frac{d}{dx}+V(x)\right\}\delta B_x(x)=0}$$

[C&H, 1974]

$\Rightarrow$ at $x=x_0$: $\omega^2=\omega_A^2(x_0)\Rightarrow$ SAW resonance

$\Rightarrow$ singularity and resonant wave absorption

◎ "Singularity" in the ideal MHD (magnetohydrodynamic) theory (valid for macroscopic dynamics)

$\Rightarrow$ importance of microscopic scale lengths :

$\rho_i = V_{ti}/\Omega_i$ (ion Larmor radius); $\rho_s=\frac{C_s}{\Omega_i}$, $C_s=V_{ti}\sqrt{\frac{T_e}{T_i}}$

$\Rightarrow$ Discovery of Kinetic Alfvén Wave (KAW) [1975, 1976]

$\Rightarrow$ complicated analyses : $|\omega/\Omega_i|\ll 1$ limit of Vlasov dynamics $\Rightarrow$ intractable for realistic nonuniform plasmas / in the nonlinear regime

$\Rightarrow$ Employing the powerful gyrokinetic theory to study KAW physics !!

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(II) Linear Kinetic Physics

(II.A) Linear gyrokinetic theory [1978; cf. JGR 1991]

◎ Theoretical foundation –

- $\epsilon \equiv \rho_i / a$; a : macroscopic scale, $\epsilon \sim O(10^{-3} - 10^{-2}) \ll 1$
- $\left|\frac{\omega}{\Omega_i}\right| \sim O(\epsilon)$, Ω_i : cyclotron freq., $|k_\perp \rho_i| \sim O(1)$
- $|\omega| \sim |k_\parallel v_t|$ (wave-particle "Landau" resonance)

$\Rightarrow |k_\parallel / k_\perp| \sim O(\epsilon)$; $|\omega| / |k_\perp V_A| \sim O(\epsilon) \Rightarrow$ fast wave suppressed!

◎ Phase-space coordinate transformation –

- $(\underset{\sim}{x}, \underset{\sim}{v}) \Rightarrow$ slowly varying guiding-center variables
  $\Rightarrow$ systematically averaging out the fast cyclotron motion
- $\boxed{(\underset{\sim}{X}, \underset{\sim}{V})_g}$: $\underset{\sim}{X}_\perp = \underset{\sim}{x}_\perp + \underset{\sim}{\rho}$, $\underset{\sim}{\rho} = \underset{\sim}{v} \times \underset{\sim}{b}_0 / \Omega_c$, $X_\parallel = x_\parallel$,
  $\underset{\sim}{V} = (\varepsilon = v^2/2 , \mu = v_\perp^2 / 2B_0)$; μ : adiabatic inv.

◎ Consider the simple case:

- uniform, isotropic Maxwellian plasma
- particle velocity distribution function :

$$\boxed{f(\underset{\sim}{x}, \underset{\sim}{v}, t) = F_M(\varepsilon) + \delta f(\underset{\sim}{x}, \underset{\sim}{v}, t)}$$

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③ field variables : $(\delta\phi, \delta\underset{\sim}{A}) + \underset{\sim}{\nabla}\cdot\delta\underset{\sim}{A} = 0$

$\Rightarrow (\delta\phi, \delta A_{\parallel}, \delta B_{\parallel})$: $\delta B_{\parallel} \simeq 0$ in $\beta \left(\equiv \frac{4\pi n_0 T}{B_0^2}\right) \ll 1$ plasmas

(i) $$\boxed{\delta f = -\frac{q}{T} F_M \delta\phi + e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} \delta g}$$ $f_0 = F_{Maxwellian}$

$\underset{\sim}{\rho} = \underset{\sim}{v} \times \underset{\sim}{b} / \Omega_c$

(ii) $$\boxed{(\partial_t + v_{\parallel}\nabla_{\parallel})\delta g = \frac{q}{T} F_M \partial_t \langle \delta L_g \rangle_{\alpha}}$$

(iii) $$\langle \delta L_g \rangle_{\alpha} = \left\langle e^{\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} (\delta\phi - v_{\parallel}\delta A_{\parallel}/c) \right\rangle_{\alpha}$$ gyrophase-averaging

(iv) $$\delta g_{j\underset{\sim}{k}} = -\left[\frac{e}{T} F_M J_0(k_{\perp}\rho) \frac{\omega}{k_{\parallel}v_{\parallel} - \omega} (\delta\phi - v_{\parallel}\delta A_{\parallel}/c)_{\underset{\sim}{k}}\right]_j$$

(v) let $\boxed{\delta\psi_{\underset{\sim}{k}} = (\omega\delta A_{\parallel}/c k_{\parallel})}$ effective induced potential

$(\delta\phi, \delta\underset{\sim}{A} = \delta A_{\parallel}\underset{\sim}{b})$

Field Eqns.

(vi) Quasi-neutrality condition

$$\boxed{\sum_j \left(\frac{n_0 e^2}{T}\right)_j \delta\phi_{\underset{\sim}{k}} + \sum_j \left(\frac{n_0 e^2}{T}\right)_j \Gamma_{0j} \left[\xi_j Z_j \delta\phi_{\underset{\sim}{k}} - (1 + \xi_j Z_j)\delta\psi_{\underset{\sim}{k}}\right] = 0,}$$

- $\Gamma_{0j} = I_0(k_{\perp}\rho_j)\exp(-k_{\perp}^2\rho_j^2/2 \equiv b_j) = I_0(b)\exp(-b)$
- $\xi_j = \omega/|k_{\parallel}v_t|_j$; $Z(\xi)$: plasma function

(vii) Vorticity eqn. $\boxed{\underset{\sim}{\nabla}\cdot\delta\underset{\sim}{J} = \nabla_{\parallel}\delta J_{\parallel} + \underset{\sim}{\nabla}_{\perp}\cdot\delta\underset{\sim}{J}_{\perp} = 0}$ → Ampere's Law

$$\boxed{\left[i\frac{c^2}{4\pi} k_{\perp}^2 \frac{k_{\parallel}^2}{\omega_{\underset{\sim}{k}}}\right]\delta\psi_{\underset{\sim}{k}} - i\omega_{\underset{\sim}{k}} \delta\phi_{\underset{\sim}{k}} \left[\sum_j \left(\frac{n_0 e^2}{T}\right)_j (1 - \Gamma_0)_j\right] = 0}$$

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(viii) $|k_\perp^2 \rho_e^2| \ll 1$, vorticity eqn. $\Rightarrow$

$$\Rightarrow \boxed{\omega_k^2 \delta\phi_k = \delta\psi_k (k_\parallel V_A)^2 b_k / (1-\Gamma_k)}.$$

$b_k = b_i = k_\perp^2 \rho_i^2 / 2$

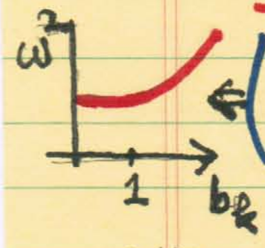
$\Gamma_k = \Gamma_{0i}$

(ix) $|\xi_i| = |\omega / k_\parallel v_{ti}| \gg 1 \gg |\xi_e| = |\omega / k_\parallel v_{te}| \Leftarrow \boxed{1 \gg \beta_i, \beta_e \gg m_e/m_i}$

$$\text{Q-N} \Rightarrow \boxed{\delta\psi_k = \left[1 + \left(\frac{T_e}{T_i}\right)(1-\Gamma_k)\right] \delta\phi_k \equiv \sigma_k \delta\phi_k}$$

$\frac{T_e}{T_i} \equiv \tau$

(x) Linear dispersion relation



$$\Leftarrow \boxed{\omega_k^2 \simeq (k_\parallel V_A)^2 \sigma_k b_k / (1-\Gamma_k)}$$

recovers previous Vlasov results

(xi) finite parallel electric field

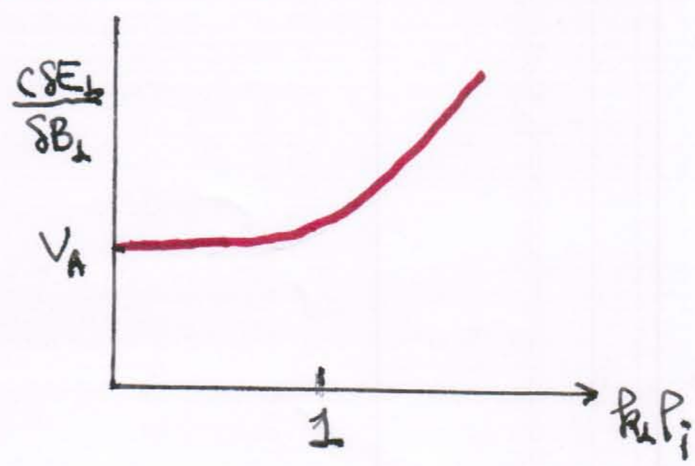
$$\boxed{\delta E_{\parallel k} = i k_\parallel \tau (1-\Gamma_k) \delta\phi_k.} \simeq i k_\parallel \begin{bmatrix} k_\perp^2 \rho_i^2 \tau \\ \tau \end{bmatrix} \delta\phi_k \quad \begin{matrix} b_k \ll 1 \\ b_k \gg 1 \end{matrix}$$

$\Rightarrow$ accellerating/heating of charged particles

aurorae, solar corona

(x) Wave polarization

$$\frac{c\,\delta E_\perp}{\delta B_\perp} = V_A \left[\frac{b_k}{\sigma_k (1-\Gamma_k)}\right]^{1/2}$$



$\Rightarrow$ important for wave identification

Non-Uniform Plasma

◎ $|k_\perp^2 \rho_i^2| \ll 1$ limit :

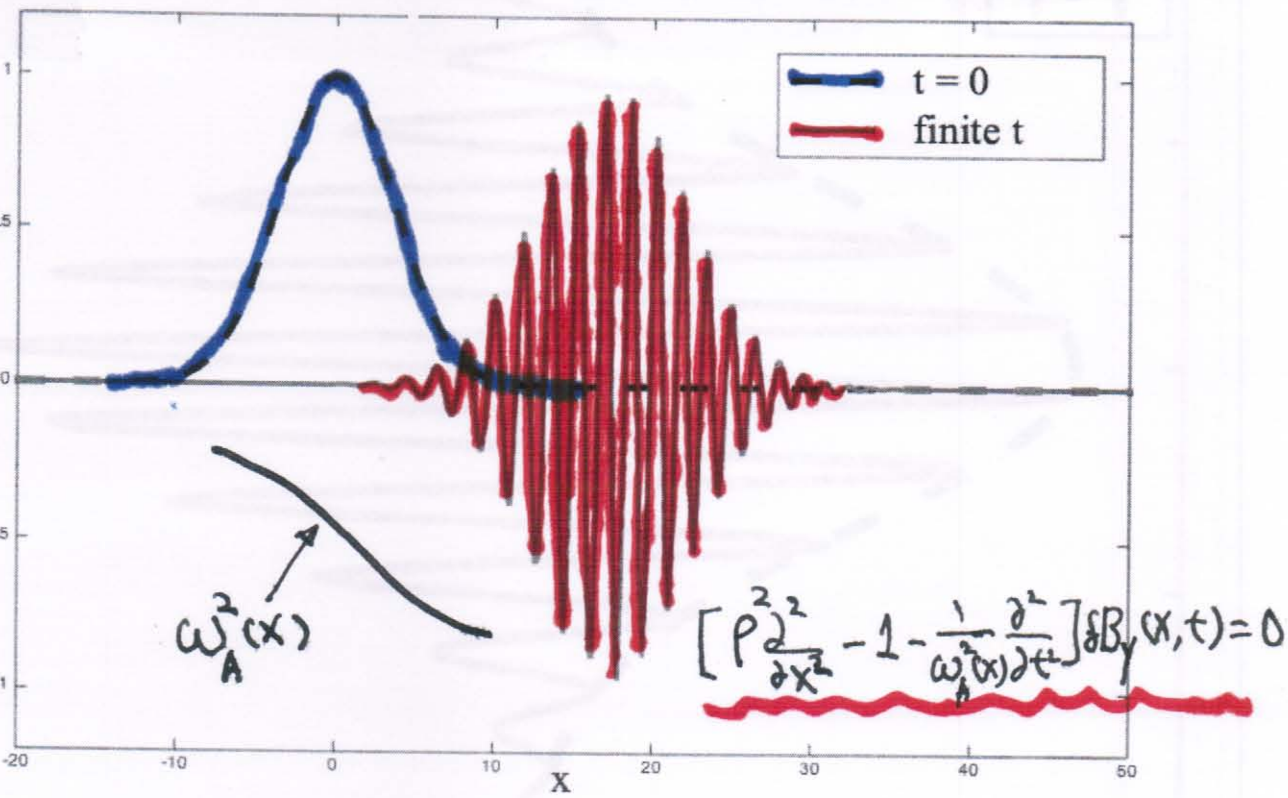
$\Rightarrow$ $\boxed{\omega_k^2 \simeq \omega_A^2(x)\left[1 + k_\perp^2 \rho^2\right]}$; $\rho^2 \equiv \rho_i^2\left[\frac{3}{4} + \frac{T_e}{T_i}\right]$

$\Rightarrow$ $k_\perp^2 > 0$, $\omega^2 > \omega_A^2(x) \Rightarrow$ propagating

$k_\perp^2 < 0$, $\omega^2 < \omega_A^2(x) \Rightarrow$ cut off

$\Rightarrow$ finite group velocity $\perp \underset{\sim}{B}_0$

$\underset{\sim}{V}_{g\perp} \cong \frac{\omega_A}{k_\perp}\left(k_\perp^2 \rho^2\right) \hat{\underset{\sim}{k}}_\perp$



[Z. Qiu thesis] GAM ⇒ KGAM

⊙ Steady state ("singular" resonance) ⇒ mode conversion

$$\Rightarrow \boxed{\left\{\rho^2 \frac{d^2}{dx^2} + \left[\frac{\omega_0^2}{\omega_A^2(x)} - 1\right]\right\} \delta\hat{B}_y(x) = \delta\hat{B}_{y0} \quad [1976]}$$

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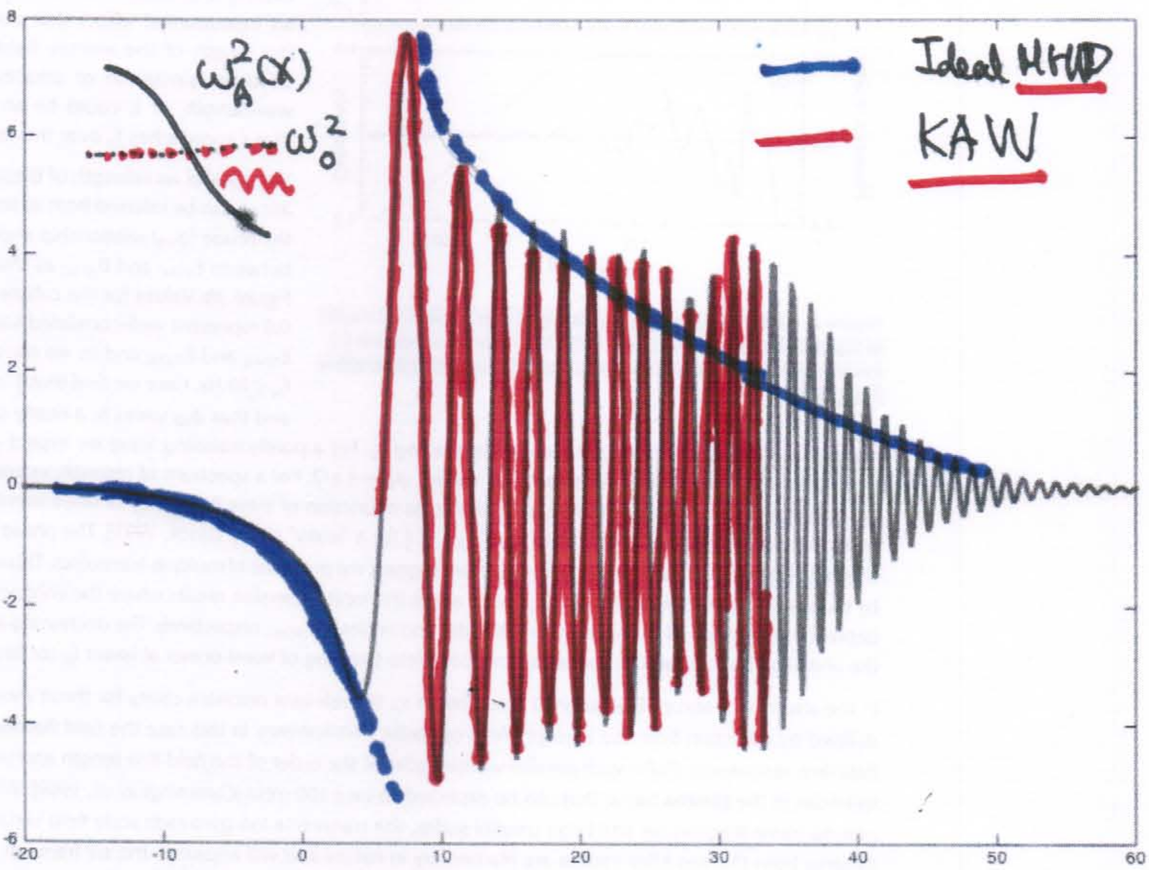

$\Rightarrow$ Scale length : $\boxed{(\Delta x)_0 = (\rho^2 L_A)^{1/3}}$, L_A : Scale of $\omega_A^2(x)$

$\Rightarrow$ temporal scale : $|k_x|_0 = \omega_A' t_0 = (\Delta x_0)^{-1}$

$$\Rightarrow \boxed{\omega_0 t_0 = \left(\frac{L_A}{\rho}\right)^{2/3} \sim O(10^2)}$$

$\Rightarrow$ "Airy" enhancement :

$$|\delta\hat{B}_y(\Delta_x)| \simeq O\left[(L_A/\rho)^{2/3}\right] |\delta\hat{B}_{y0}|$$

Van Allen Probes [Chaston et al 2014] GRL P11

Inner Magnetosphere (L ~ 6 R_E)

Satellite observations

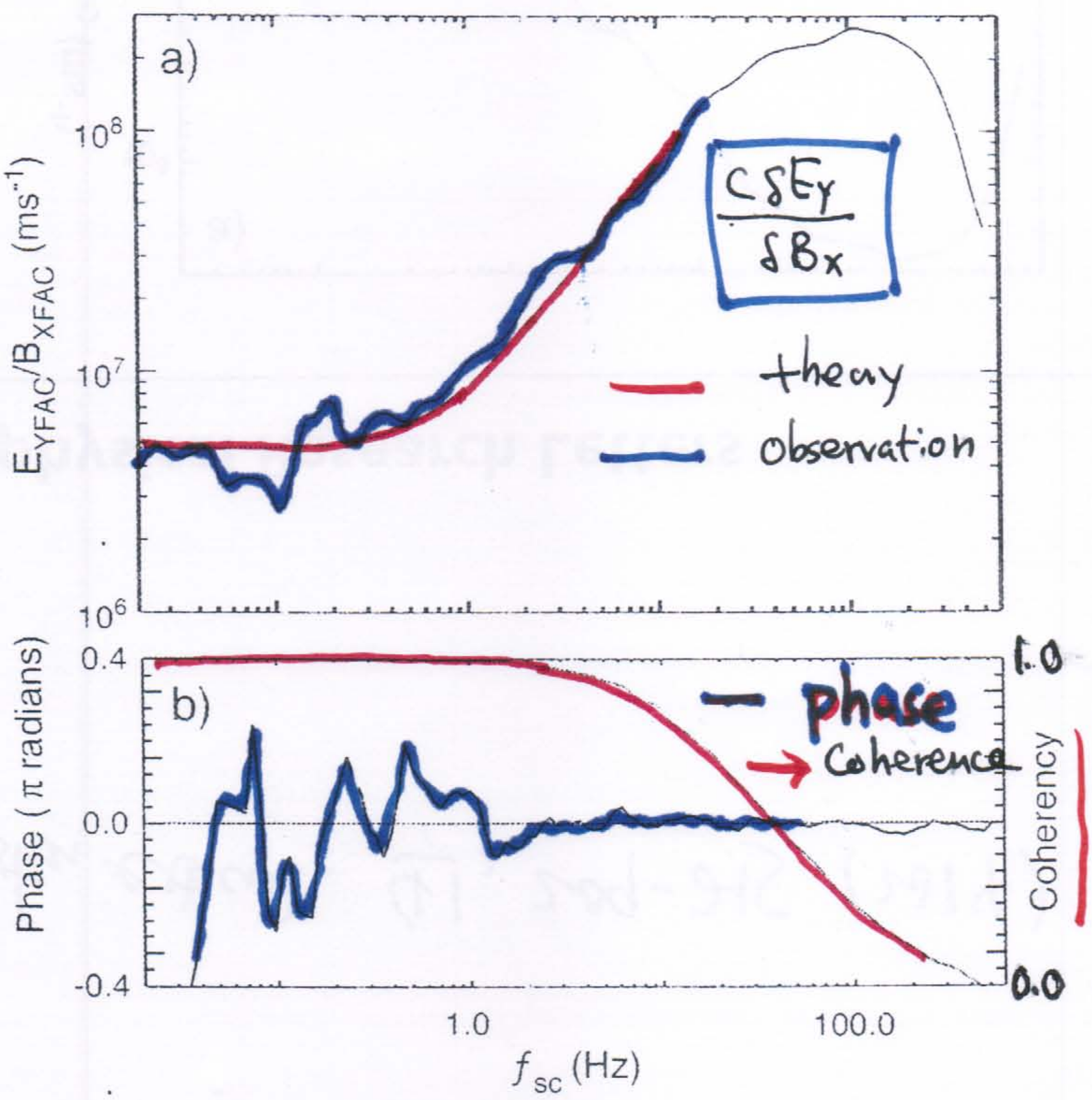

Figure 3. (a) The ratio E_{YFAC}/B_{XFAC} averaged over the interval shown in Figure 2. Red line shows the fit of the local dispersion relation for kinetic Alfvén waves. (b) Relative phase and coherency (red) between E_{YFAC} and B_{XFAC}.

$k_\perp \rho_i \gtrsim 1$ and in
Doppler shift
mentioned abo
will be larger t
frequency (f_p) f
accounts for the
though for the
The origin of
B_{XFAC} ratio for
writing is uncle
an instrumenta
the length of t
becomes simila
wavelength, or
that f approach

The parallel wav
ations can be inf
the phase (ϕ_{EB})
between E_{YFAC} a
Figure 3b. Value
0.6 represent w
E_{YFAC} and B_{XFAC}
$f_{sc} \leq 20$ Hz. Here
and that ϕ_{EB} va

manner as it decreases in magnitude with increasing f_{sc}. For a purely traveling w
while for a standing wave or resonance we expect $\phi_{EB} = \pm\pi/2$. For a spectrum o
multiple harmonics he expected phase variation as a function of wave frequenc
latory between $\pm\frac{\pi}{2}$ for a perfect cavity and $|\phi_{EB}| < \frac{\pi}{2}$ for a "leaky" cavity [*Lysak*, 1

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Salam et al

Cluster observations in solar wind [ApJ 2012]

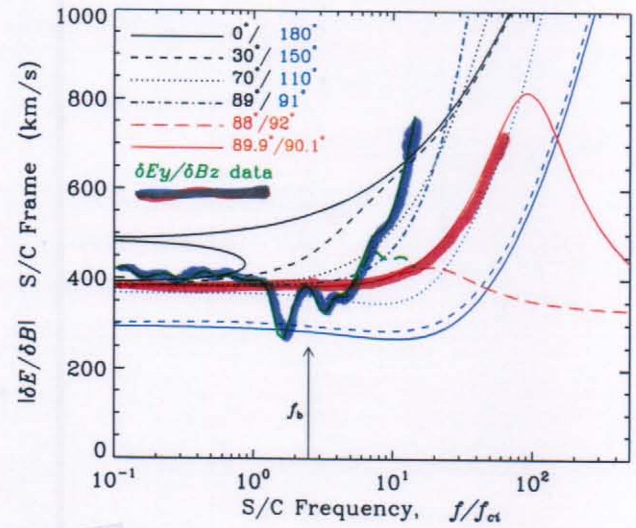

Figure 3. Prediction of $|\delta\mathbf{E}/\delta\mathbf{B}|_{s/c}$ for kinetic Alfvén waves (red curves) or whistler waves (black and blue curves) with specified angle θ. *Cluster* measurements of $|\delta E_y/\delta B_z|$ up to 2 Hz, or $12 f_{ci}$, are presented without (green solid) and with (green dashed) the EFW noise floor removed.

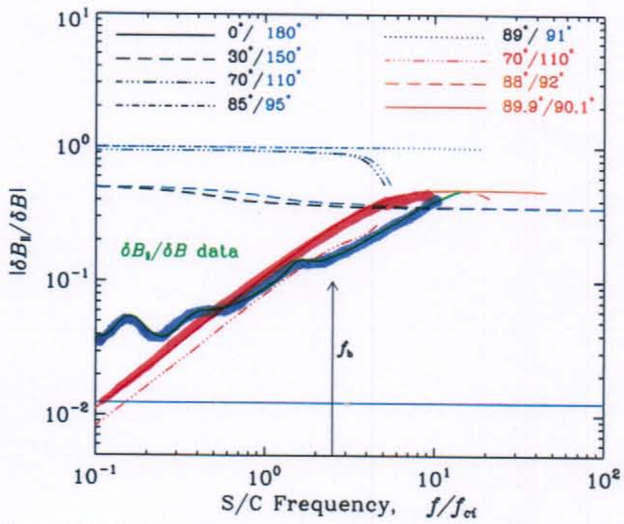

Figure 4. Prediction of $|\delta B_{\|}|/|\delta\mathbf{B}|_{s/c}$ for kinetic Alfvén waves (red) or whistler waves (black/blue) with specified angle θ. *Cluster* FGM measurements up to 2 Hz, or $12 f_{ci}$, are shown in green.

(A color version of this figure is available in the online journal.)

$\boxed{c\,\delta E_\perp/\delta B_\perp}$

— observation

— theory

$\boxed{\delta B_\parallel/\delta B}$

— observation

— theory

ⓓ Lab observations:

- ~1980 Lausanne, Swiss
- TFTR [K.L. Wong]

(III) Nonlinear Kinetic Physics

(III.A) Nonlinear gyrokinetic theory [cf. 1982] F-C

◎ Nonlinear ordering :

$$|\delta f / F_0| \sim O(\epsilon) \Rightarrow |\delta \underset{\sim}{u}_\perp \cdot \underset{\sim}{\nabla}_\perp| \equiv \omega_{nl} \sim O(\omega)$$

$\Rightarrow$ Valid for strong turbulence

◎ Nonlinear gyrokinetic equations

- $\boxed{\delta f = -\frac{q}{T} F_M \delta\phi + e^{-\underset{\sim}{\rho} \cdot \underset{\sim}{\nabla}} \delta g}$
- $\boxed{[\partial_t + v_\parallel \partial_\ell + \langle \delta \underset{\sim}{u}_{Eg} \rangle \cdot \underset{\sim}{\nabla}] \delta g = \frac{q}{T} F_M \partial_t \langle \delta L_g \rangle_\alpha}$
- $\boxed{\langle \delta L_g \rangle = \langle e^{\underset{\sim}{\rho} \cdot \underset{\sim}{\nabla}} (\delta\phi - v_\parallel \delta A_\parallel / c) \rangle_\alpha}$ gyrophase averaging
- $\boxed{\langle \delta \underset{\sim}{u}_{Eg} \rangle = \frac{c}{B_0} \underset{\sim}{b} \times \underset{\sim}{\nabla} \langle \delta L_g \rangle_\alpha}$

$$= \frac{c}{B_0} \langle \delta \underset{\sim}{E}_\perp \rangle_\alpha \times \underset{\sim}{b} + v_\parallel \frac{\langle \delta \underset{\sim}{B}_\perp \rangle_\alpha}{B_0}$$

* $\beta \ll 1 \Rightarrow |\delta B_\parallel| \simeq 0$

- Fourier expansion $\Rightarrow$

$$\boxed{i(k_\parallel v_\parallel - \omega_k)\delta g_k = -i\frac{\omega_k q}{T} J_k \delta L_{gk} F_M + \frac{c}{B_0}\Lambda_{k'}^{k''}\left[J_{k'}\delta L_{k'}\delta g_{k''} - J_{k''}\delta L_{k''}\delta g_{k'}\right]}$$

- $\boxed{\Lambda_{k'}^{k''} = \underset{\sim}{b}\cdot(\underset{\sim}{k}_\perp' \times \underset{\sim}{k}_\perp'')}$, $\underset{\sim}{k}' + \underset{\sim}{k}'' = \underset{\sim}{k}$ (*)

- $\delta L_k = \delta\phi_k - v_\parallel \delta A_{\parallel k}/c$

Field equations

- quasi-neutrality condition ($q_i = e$)

$$\Rightarrow \boxed{e n_0(1+\tau)\frac{\delta\phi_k}{T_e} = \langle J_k \delta g_{ki} - \delta g_{ke}\rangle_v}$$

- $\tau \equiv T_e/T_i$ $\Leftarrow$ Poisson's eqn $|k\lambda_D|^2 \ll 1$

- Parallel Ampere's Law

$$\Rightarrow \quad k_\perp^2 \delta A_{\parallel k} = \frac{4\pi}{c}\delta J_{\parallel k}$$

$$+ \ \underset{\sim}{\nabla}\cdot\delta\underset{\sim}{J} = i k_\parallel \delta J_{\parallel k} + i\underset{\sim}{k}_\perp\cdot\delta\underset{\sim}{J}_{\perp k} = 0 \Rightarrow$$

$$\sum_j q_j \langle J_k (*)\rangle_{jv} \Rightarrow$$

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[RMP 2016]

③ The nonlinear gyrokinetic vorticity eqn.

line-bending inertial

$$\Rightarrow \boxed{i k_\parallel \delta J_{\parallel k} - i \frac{c}{4\pi} \frac{\omega_k}{V_A^2} \frac{k_\perp^2}{b_k} (1-\Gamma_k)\, \delta\phi_k = (NL)_A + (NL)_\phi}$$

- $b_k = k_\perp^2 \rho_i^2/2, \quad \Gamma_k = I_0(b_k)\exp(-b_k)$
- $(NL)_A = -\Lambda_{k'}^{k''} [\delta A_{\parallel k'} \delta J_{\parallel k''} - \delta A_{\parallel k''} \delta J_{\parallel k'}]/B_0$

$\Rightarrow$ "Maxwell stress"

- $(NL)_\phi = \frac{ec}{B_0} \Lambda_{k'}^{k''} \Big\langle (J_k J_{k'} - J_{k''}) \delta L_{k'} \delta g_{k''} - (J_k J_{k''} - J_{k'}) \delta L_{k''} \delta g_{k'} \Big\rangle_i$

$\Rightarrow$ "gyrokinetic-ion Reynold stress"

(III.B) Parametric decay instabilities

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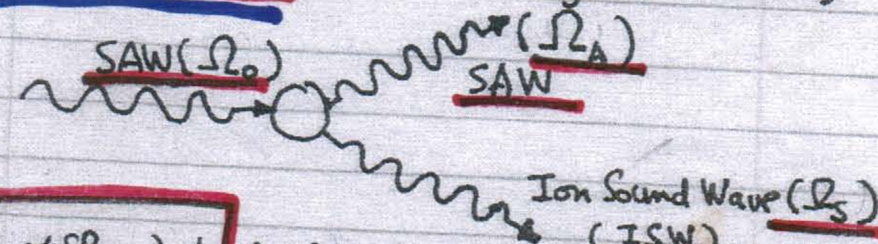
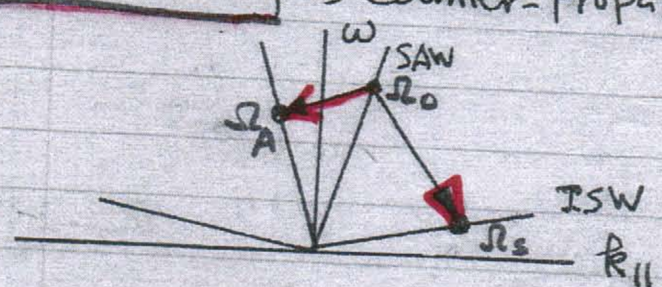
3-wave parametric decays

◎ **Ideal MHD theories** (1969 Sagdeev & Galeev)



- $\boxed{(\delta \underset{\sim}{J}_{\perp A} \times \delta \underset{\sim}{B}_{\perp A}) \cdot \underset{\sim}{b}} \Rightarrow \delta u_{\parallel S} \Rightarrow \delta n_S$
- $\boxed{\delta n_S \, \delta \underset{\sim}{u}_{PA}} \Rightarrow \delta \underset{\sim}{J}_{\perp A}^{ne}$
- Backscattering ⇒ Counter-Propagating SAWs

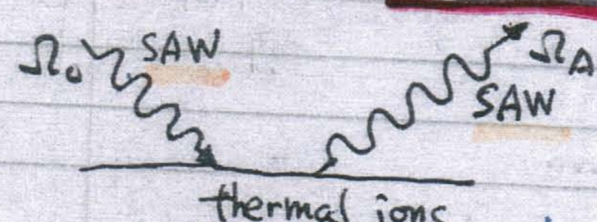


- ω-$\underset{\sim}{k}$ matching conditions

$$\Omega_0 = (\omega_0, \underset{\sim}{k}_0) = \Omega_S + \Omega_A$$

◎ Similar physics for nonlinear ion Landau damping

$T_e \sim T_i$:



ion-induced scatterings

- $\omega_S = \omega_0 - \omega_A \approx k_S v_{ti}$

  ⇒ Ω_S heavily damped ISW (quasi mode)

P.17

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GK Theory of Parametric Decays of KAW

(2011)

overview of results

(I) Parametric dispersion relation

$$\boxed{\epsilon_s [\epsilon_{A-} + \chi_{A-}^{(2)}] = C_K |\Phi_0|^2}, \quad |\Phi_0| : \text{pump wave}$$

- s : low-freq. sound wave

ISW $\boxed{\epsilon_s = 1 + \tau + \tau \Gamma_s \xi_s Z(\xi_s)}$, $\tau = T_e/T_i$,

$\Gamma_s = I_0(b_{is}) \exp(-b_{is})$,

$\xi_s = |\omega_s| / |k_{s\parallel}| v_{ti}$

$b_k = k_\perp^2 \rho_i^2 / 2$

- A_- : Daughter KAW

KAW $\boxed{\epsilon_{A-} = \frac{(1-\Gamma_-)}{b_{i-}} - \left(\frac{k_\parallel V_A}{\omega}\right)_-^2 \sigma_-}$; $\sigma_- = 1 + \tau(1-\Gamma_-)$

- C_K : Shielded Ion Scatterings

- $\boxed{C_K = \lambda^2 H^2}$, $\sim O(|\frac{\Omega_i}{\omega_0}|^2) \gg 1$
- $\lambda^2 = \left(\frac{\Omega_i}{\omega_0}\right)^2 \left[(\underline{k}_s \rho_s \times \underline{k}_0 \rho_s) \cdot \underline{b}\right]^2 / (b_{s-} \sigma_-) \sim O(\frac{\Omega_i^2}{\omega_0^2}) \gg 1$
- $\boxed{H = (\sigma_0 \sigma_- - F_1 \sigma_s / \Gamma_s)} \sim O(1)$
- $F_1 = \int d^3v\, F_{Mi} J_0 J_- J_s \equiv \langle J_0 J_- J_s \rangle_v$

- χ_{A-} : Bare ion scatterings (Ion Compton Scattering)

$\boxed{\chi_{A-} = \epsilon_s (\lambda^2 / \Gamma_s) G |\Phi_0|^2}$; $G = [\langle J_0^2 J_-^2 \rangle_v - F_1^2 / \Gamma_s] \geq 0$

Schwartz inequality

P-1[illegible]
2019 [illegible]

◎ $|k_\perp \rho_i|^2 \ll 1 \Rightarrow |RHS| \propto \left|\frac{\delta B_{\perp 0}}{B_0}\right|^2 (k_{\perp s}\rho_i)^4$

◎ $|k_\perp \rho_i|^2 \to \infty \Rightarrow |RHS| \propto \left|\frac{\delta B_{\perp 0}}{B_0}\right|^2 (k_{\perp s}\rho_i)^{-1}$

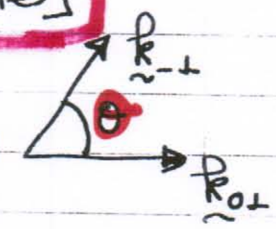
$\Rightarrow$ maximal scatterings around $|k_\perp \rho_i| \sim O(1)$.

(II) Contrast with ideal MHD theory (S+G, 1969)

◎ C_K replaced by

MHD $\Rightarrow$ $\boxed{C_I = |\underset{\sim}{k}_{0\perp}\rho_s \cdot \underset{\sim}{k}_{\perp -}\rho_s|^2 / [b_{s-}(1+\delta_i T_i/T_e)]}$

$\boxed{\Rightarrow \frac{b_{s0}}{(1+\delta_i T_i/T_e)} \cos^2\theta} \approx O(1)$



◎ $\boxed{C_K = \left(\frac{\Omega_i}{\omega_0}\right)^2 \frac{b_{s0}}{\delta_-} H^2 \sin^2\theta} \approx O\left(\frac{\Omega_i}{\omega_0}\right)^2$

$\Rightarrow \boxed{|C_K|/|C_I| \sim O\left(\frac{\Omega_i}{\omega_0}\right)^2 \ggg 1 \text{ for } |k_\perp \rho_i| \sim O(1)}$

◎ For $|k_\perp \rho_i|^2 \ll 1$

* $\Rightarrow |C_K|/|C_I| \sim O\left(\frac{\Omega_i}{\omega_0}\right)^2 (k_\perp \rho_i)^4$

* $\Rightarrow \boxed{|k_\perp \rho_i| \gtrsim O\left(\frac{\omega_0}{\Omega_i}\right)^{1/2} \ll 1 \text{ kinetic effects dominate!!}}$

$\Rightarrow$ ◎ ideal MHD approximation breaks down faster in the nonlinear physics regime quantitatively and qualitatively !!

(following discussions)

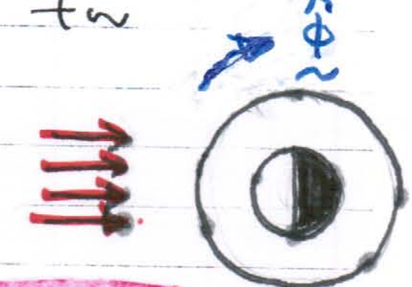
◎ Qualitative difference

- $C_I \propto \cos^2\theta \Rightarrow$ maximizes for

MHD $\Rightarrow \boxed{\underset{\sim}{k}_{\perp -} \parallel \underset{\sim}{k}_{\perp 0}}$

- $C_K \propto \sin^2\theta \Rightarrow$ maximizes for

Kinetic $\Rightarrow \boxed{\underset{\sim}{k}_{\perp -} \perp \underset{\sim}{k}_{\perp 0}}$



- Mode conversion $\Rightarrow \boxed{\underset{\sim}{k}_{\perp 0} = k_{\perp 0}\, \hat{\underset{\sim}{r}}}$

MHD $\Rightarrow C_I \Rightarrow \underset{\sim}{k}_{\perp -} \approx k_{\perp -}\, \hat{\underset{\sim}{r}} \Rightarrow$ little transport!! **

Kinetic $\Rightarrow C_K \Rightarrow \underset{\sim}{k}_{\perp -} \approx k_{\perp -}\, \hat{\underset{\sim}{\phi}} \Rightarrow$ symmetry breaking

$\Rightarrow$ finite transport!!

- $C_I \Rightarrow$ anisotropic spectrum $\perp \underset{\sim}{B}_0$.

  $C_K \Rightarrow$ isotropic spectrum $\perp \underset{\sim}{B}_0$.

** [Note: $D_{rr} \propto k_\phi^2 |\delta\phi|^2$] (JGR 1999)

◎ generalized angular momentum

$\langle P_\phi \rangle_\alpha \Leftrightarrow$ guiding-center position
radial

R20 h

Lin et. al. PRL [2012]

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Hybrid simulation

Mode Conversion KAW

$|k_x| \gg |k_y|$ pump

$|k_y| \gg |k_x|$ decay KAW

$T_i/T_e \cong 2.5$

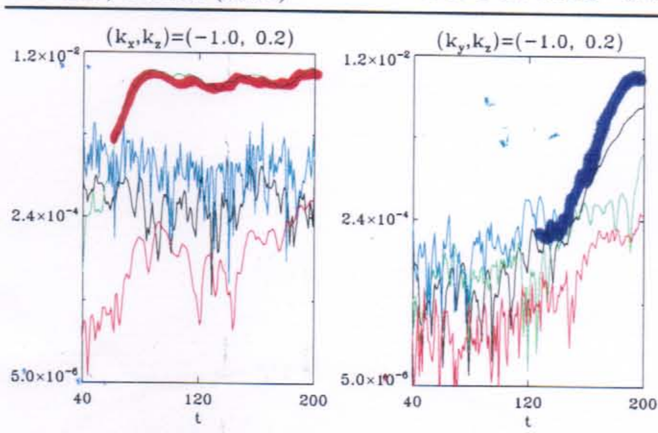


FIG. 3 (color). Time evolution of B_x (black), B_y (green), E_y (blue), and $E_\parallel$ (red) for the KAW modes dominated by k_x (left column) at $y = 32$ and for those dominated by k_y (right) at the resonance point $x = 53$ in the inhomogeneous magnetopause.

B_x $E_\parallel$ → pump

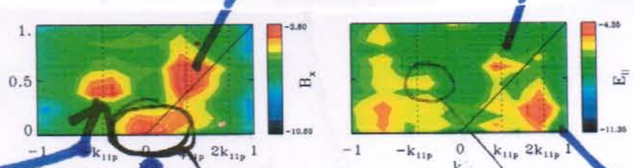
uniform pump

3

ω

ISW

decay KAW



$k_\parallel$

FIG. 4 (color online). $k_\parallel$-ω spectra of B_x and $E_\parallel$ obtained from the simulation of decay of an initial pump KAW in a uniform plasma. The solid black line indicates the dispersion relation of the MHD shear Alfvén mode for reference.

→ decay KAW

$k_\parallel \approx 0$

→ MSCC [Convective cells later]

∘ $|k_y \rho_i| \sim O(1)$ maximal PDI

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(III.C) ⊙ $\boxed{\text{Convective cells } (\sim 1970\text{'s})}$

$\Rightarrow$ $\boxed{\underset{\sim}{k} \cdot \underset{\sim}{B}_0 = 0}$ and $\boxed{\omega \simeq 0}$

- Electrostatic convective cells (ESCC)

  $\Rightarrow$ $\delta \underset{\sim}{E} = \delta \underset{\sim}{E}_\perp$ [Dawson and Okuda]

- Magnetostatic convective cells (MSCC)

  $\Rightarrow$ $\delta \underset{\sim}{B} = \delta \underset{\sim}{B}_\perp$ [Chu et al]

$\Rightarrow$ anomalous $\perp \underset{\sim}{B}_0$ transport.

⊙ In fusion laboratory plasmas –

$\boxed{[\underset{\sim}{k} \cdot \underset{\sim}{B}_0 = 0]}$

Zonal structures $\Rightarrow$ self-regulation of turbulence (transport)

- Zonal flow $\Leftrightarrow$ ESCC
- Zonal field/current $\Leftrightarrow$ MSCC

$\Rightarrow$ Zonal structures : Subset of convective cells

$\Rightarrow$ Renewed interest in nonlinear excitations of convective cells !!

⊙ Convective cells generation by Alfvén waves

- In uniform, magnetized plasmas

⇒ Ideal MHD limit ⇒

Reynold stress + Maxwell stress ≃ 0

⇒ Pure Alfvénic state ⇒ no convective cell generation

- Non-ideal effects ⇒ Kinetic Alfvén Waves (KAW)

- Convective cells by KAW :

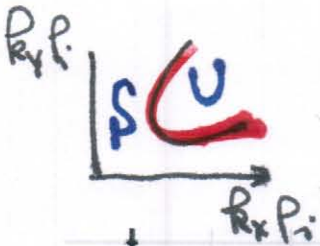
Previous studies (since 1978 by Sagdeev et al) assume $|k_\perp^2 \rho_i^2| \ll 1$ (drift-kinetic ions) and/or decoupled ESCC and MSCC ⇒ incorrect stability analyses !

- Spontaneous excitation of coupled ESCC + MSCC via modulational instability of KAW sets in when $|k_\perp \rho_i| \sim O(1)$!!

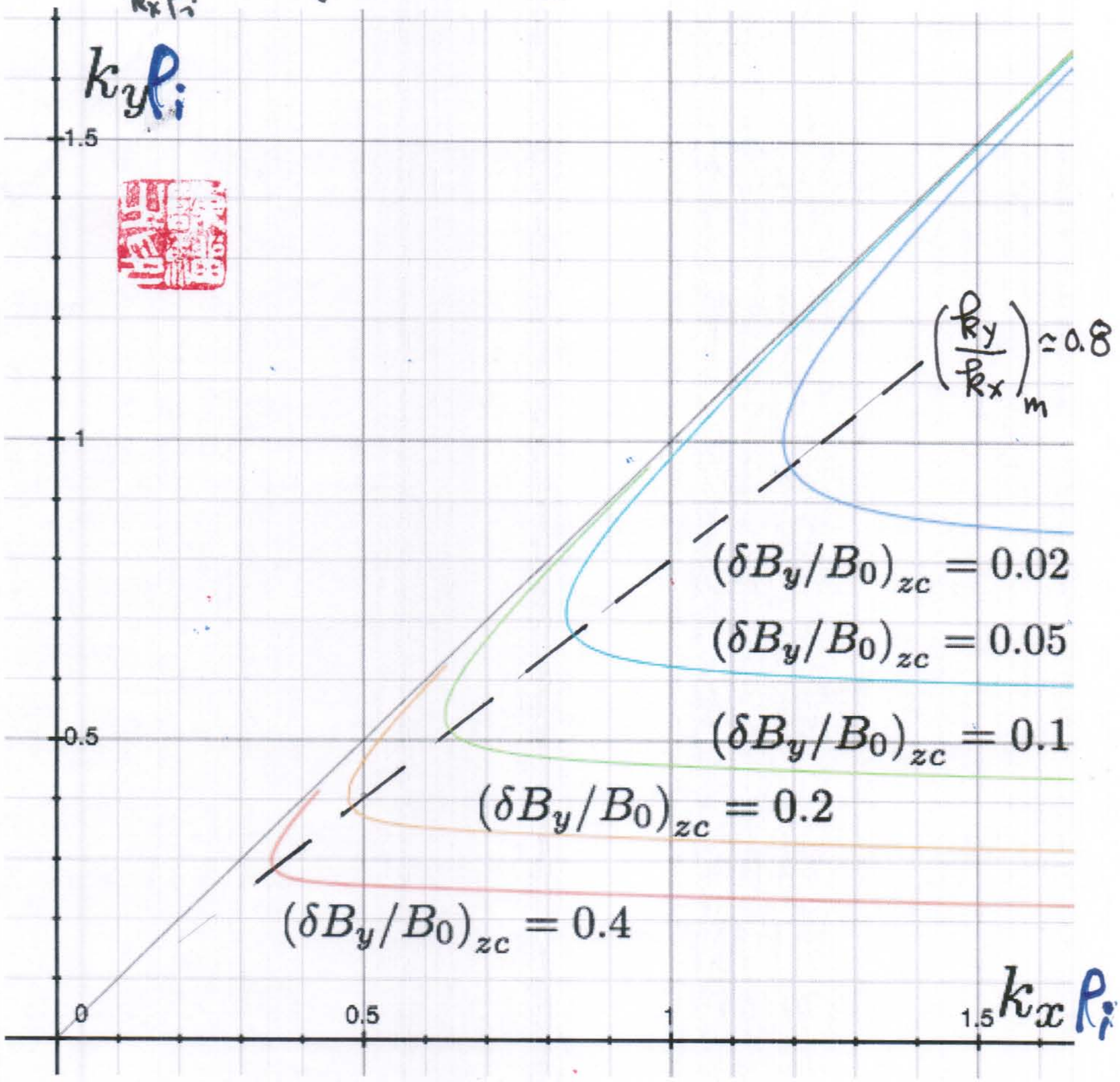
[ZLC, EPL, 2015]

Numerical Solutions of the M. I. Disp Relation

[$\beta_e = \beta_i = 0.2$, $\tau = 1$]

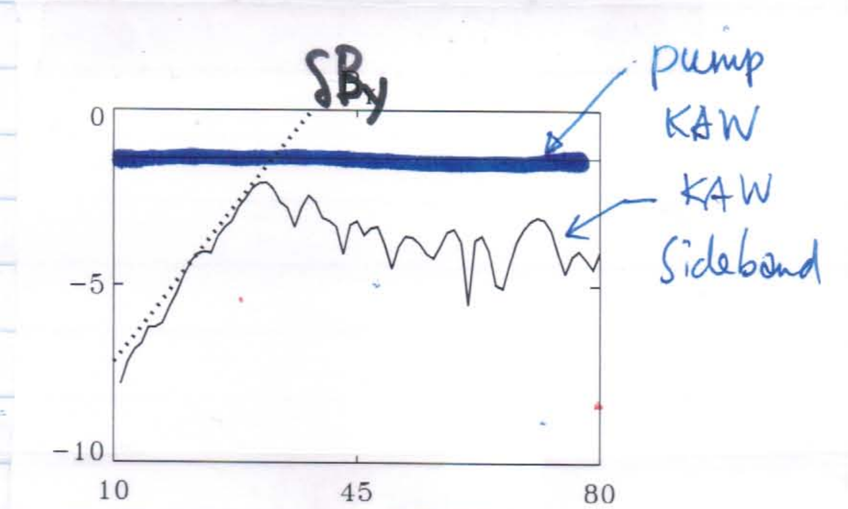


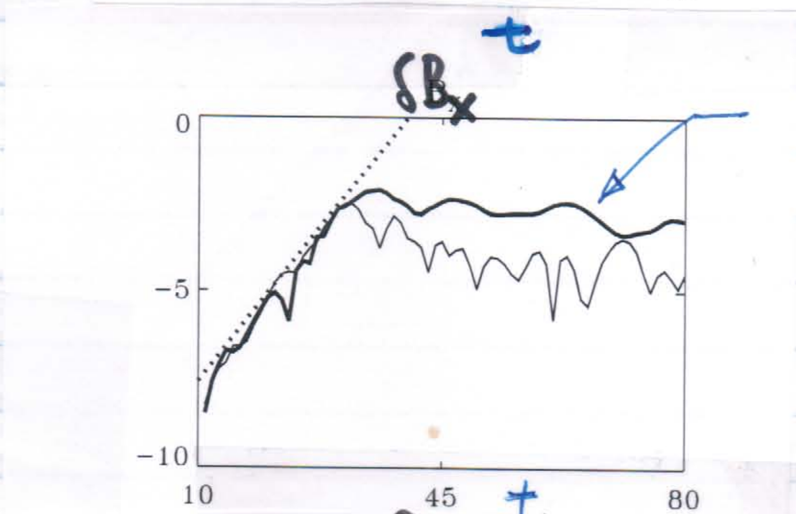
Marginal stability curves.

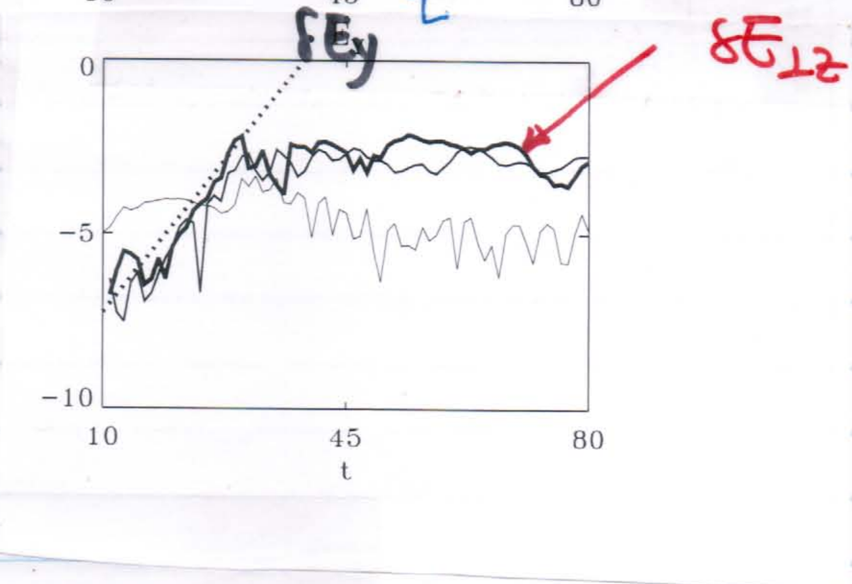


$\delta B_{y0}/B_0 = 0.4$, $k_{x0}\rho_i = 1.25$, $k_y \rho_i = 0.8$ p24

Simulation







$\Rightarrow$ MSCC (δB_{xz}) and ESCC (δE_{yz}) coupled !!

p.15 $\Rightarrow [V_t : \delta B_{xz}/c\,\delta E_{yz}] \approx 0.17$ (Theory : 0.14) p.16

P.25 加
附图

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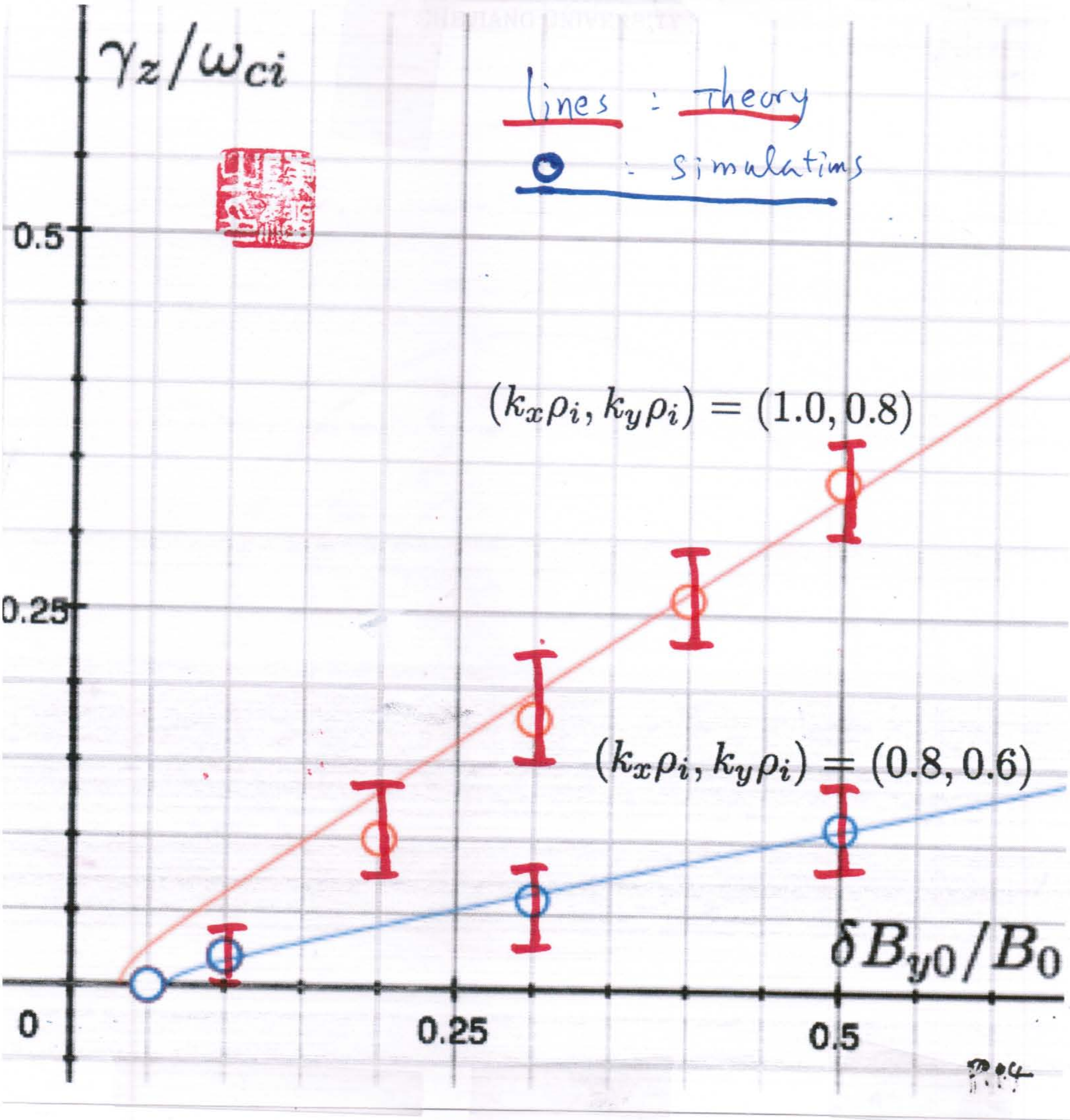

γ_z/ω_{ci}
lines : Theory
: simulations
$(k_x\rho_i, k_y\rho_i) = (1.0, 0.8)$
$(k_x\rho_i, k_y\rho_i) = (0.8, 0.6)$
$\delta B_{y0}/B_0$
0.5
0.25
0
0.25
0.5
P.14

(IV) Summary + Discussions

◎ Kinetic Alfvén waves play crucial roles in the phase-space dynamics of charged particles; accellerating, heating and transports, in nature + Lab. plasmas. [KTAE, KBAE, etc]

◎ Gyrokinetic theory provides powerful theoretical (analytical and numerical) tools for studying the physics of KAW.

◎ Finite-$k_\perp \rho_i$ effects affect both qualitatively and quantitatively the processes of parametric decay instabilities as well as the excitations of convective cells (zonal flow/current-field):
maximum scatterings occur around $|k_\perp \rho_i| \sim O(1)$

⇒ ideal MHD / 2-fluid / drift-kinetic approaches: inadequate !!

⇒ Supported by both analytical and simulation studies.

⇒ e-i decoupling much enhanced in the KAW regime!

◎ Characteristic KAW time scale:
$\omega_0 t_0 = (L_A/\rho)^{2/3} \sim O(10^2)$ and $(\gamma/\omega_A)_{AE} \sim O(10^{-2})$
for Alfvén-wave instabilities in laboratory fusion plasmas $\Rightarrow$ $\gamma_{AE} t_0 \sim O(1)$: KAW could play important roles in instabilities driven by, e.s., fusion α particles.

◎ Interesting and important issues; such as long-time-scale nonlinear dynomis in realistic non-uniform plasma enviromments remain little explored. [e.g., K-AEs & Drift Waves, etc.]

Thanks! 谢谢!!